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Monte Carlo study of the Potts glass with nearest-neighbour random Gaussian interaction

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Abstract. The three-state Potts model with nearest-neighbour random Gaussian interaction on the simple cubic lattice is investigated by Monte Carlo simulation. Both static quantities (e.g. glass ordering susceptibility and the correlation function) and dynamic quantities (the analogue of the time-dependent Edwards-Anderson order parameter $q(t)$ for Potts spins) are obtained. As for related models for orientational glasses, it is found that $q(t)$ is consistent with the Kohlrausch law $q(t) \propto \exp[-(t/\tau)^\gamma]$ for a wide range of temperatures, with a strongly temperature-dependent exponent γ , with γ becoming very small as the temperature $T \rightarrow 0$. The relaxation time τ increases dramatically as T is lowered; thus the system could only be equilibrated for temperatures where the correlation length is rather small. Since the critical region has not been reached, it cannot be distinguished whether the critical temperature T_c is non-zero or at $T = 0$. If $T_c = 0$, the divergences of τ and the ordering susceptibility are probably exponential, i.e. the system is then at its lower critical dimensionality.

1. Introduction

The p -state Potts model (Potts 1952, Wu 1982) is a lattice model where each lattice site i carries a Potts spin S_i which can take one of the values $S_i = 1, 2, \dots, p$, and the interactions are described by the Hamiltonian

$$\mathcal{H} = - \sum_{\langle i, j \rangle} J_{ij} \delta_{S_i, S_j} \quad (1.1)$$

where $\langle i, j \rangle$ denotes a summation that runs over all nearest-neighbour pairs once, and $\delta_{\alpha\beta}$ is the Kronecker symbol. Thus a pair $\{S_i, S_j\}$ contributes an energy $-J_{ij}$ if $S_i = S_j$ and zero otherwise. The ferromagnetic Potts model ($J_{ij} = J$ independent of i, j), which reduces to the Ising model for $p = 2$, has turned out to be one of the most instructive models for the study of both first-order and second-order phase transitions in statistical thermodynamics.

Here we are concerned with the random version of (1.1), where J_{ij} is not uniformly ferromagnetic but a quenched random variable described by a Gaussian distribution with zero mean:

$$P(J_{ij}) \propto \exp[-J_{ij}^2/2(\Delta J)^2] \quad \Delta J \equiv 1. \quad (1.2)$$

While the case $p = 2$, the Ising version of the spin glass model first introduced by Edwards and Anderson (1975), has been studied extensively (see Binder and Young (1986) for a review), the model has received relatively little attention for $p > 2$. The

Potts glass with an infinite-range interaction has been studied by Elderfield and Sherrington (1983a, b, c), Lage and Erzan (1983), Elderfield (1984), Lage and Nunes (1984), Gross *et al* (1985) and Goldschmidt (1988); the short-range Potts glass has so far been studied only in the framework of a renormalisation group expansion near six dimensions (Goldschmidt 1985). This work predicted a fluctuation-induced first-order transition for $2 < p < 4$, but it is unclear whether this prediction is applicable in three dimensions. The Potts glass has also been suggested as a starting point for a theory of structural glasses and the transition from the metastable fluid to the glass state (Kree *et al* 1987, Kirkpatrick and Wolynes 1987, Kirkpatrick and Thirumalai 1988). Clearly real fluids do not have infinite-range interactions, and thus the short-range Potts glass needs to be considered. This model may also serve as a model for orientational glasses where a strong single-site anisotropy restricts the orientation of the quadrupole moment associated with the appropriate molecular group to p distinct directions. Recently we have studied the opposite case of a fully isotropic quadrupolar glass (Carmesin and Binder 1987a, b). There exists rich experimental material on orientational glasses (see Knorr (1987) for a recent review) but it is not clear whether any of the models mentioned so far are appropriate to describe these systems (Kanter and Sompolinsky 1986, Michel 1986). A study of these models may help to sort out the 'universality classes' for glass transitions.

Just as for the study of short-range spin glasses (Binder and Young 1986, Bhatt and Young 1987) one has to resort to numerical techniques to study the model defined by (1.1) and (1.2). In the present work, we perform a first Monte Carlo investigation of this model. Section 2 describes our results for the time autocorrelation function of the Potts spin variable and the relaxation time, while § 3 analyses the static properties of the model and discusses the question whether a finite-temperature glass transition occurs. Section 4 contains our conclusions.

2. The time-dependent Edwards–Anderson order parameter for the short-range Potts glass

We study simple cubic $L \times L \times L$ lattices, applying periodic boundary conditions for the case $L = 12$. For the temperature 0.35 we simulated a lattice of size $L = 18$. Considering the three-state Potts model as a limiting case of a quadrupolar model with very strong cubic anisotropy, we redefine the Hamiltonian (1.1) slightly: we define a unit vector $\mathbf{S}_i = (S_i^x, S_i^y, S_i^z)$ whose components are either zero or unity, so there are only six states permissible: $\mathbf{S}_i = (\pm 1, 0, 0)$, $(0, \pm 1, 0)$ or $(0, 0, \pm 1)$ respectively. The Hamiltonian is written (as in the p -state case)

$$\mathcal{H} = - \sum_{\langle i, j \rangle} J_{ij} \left(\sum_{\mu=1}^p S_i^\mu S_j^\mu \right)^2 - h \sum_i (S_i^z)^2. \quad (2.1)$$

Note that $(\sum_{\mu=1}^p S_i^\mu S_j^\mu)^2 = 1$ for $\mathbf{S}_i = \mathbf{S}_j$ as well as for $\mathbf{S}_i = -\mathbf{S}_j$, while in all other cases $\sum_{\mu=1}^p S_i^\mu S_j^\mu = 0$. So we have to take both states $\{\mathbf{S}_i, -\mathbf{S}_i\}$ as corresponding to one state of the Potts model. In (2.1) we also have introduced a field, h , favouring 'ferromagnetic' quadrupolar ordering, but in the actual computations we use $h = 0$ unless otherwise stated. The reason for writing the Potts Hamiltonian in this form is that the simulation programs written for the isotropic and anisotropic quadrupolar glasses (Carmesin and Binder 1987a, b, Carmesin 1988a) can then be carried over to the present problem with only minor changes.

With this notation, the Potts analogue of the time-dependent Edwards-Anderson spin glass order parameter (Binder and Young 1986) can be written as (see also Carmesin and Binder (1987a, b); N is the total number of lattice sites)

$$q(t) = \left(\frac{1}{N} \sum_i \left\langle [S_i(t') \cdot S_i(t'+t)]^2 - \frac{1}{p} \right\rangle_{t'} \right)_{av} (1-1/p)^{-1} \quad (2.2)$$

where $()_{av}$ denotes the sampling over the bond distribution $P(J_{ij})$ of equation (1.2) (typically we average over 80 individual runs with different realisations $\{J_{ij}\}$), and $\langle \rangle_{t'}$ is the time average as obtained by standard Metropolis Monte Carlo sampling (Binder 1979). Since even for completely random configurations of the Potts spins $[S_i(t') \cdot S_i(t'+t)]^2$ is unity with probability $1/p$, a term $1/p$ is subtracted in order that $q(t) = 0$ for uncorrelated Potts spins. The denominator $(1-1/p)$ is introduced so that $q(t) = 1$ for a perfectly correlated case (the corresponding denominator $(1-1/m)$ has been erroneously omitted in equation (9) of Carmesin and Binder (1987b) and in equation (3) of Carmesin and Binder (1987a)).

Figure 1 presents our numerical results for $q(t)$, plotting the same Monte Carlo 'data' in three different ways to illustrate the extent to which different decay laws can be distinguished. The decay laws considered are motivated by analogous findings for

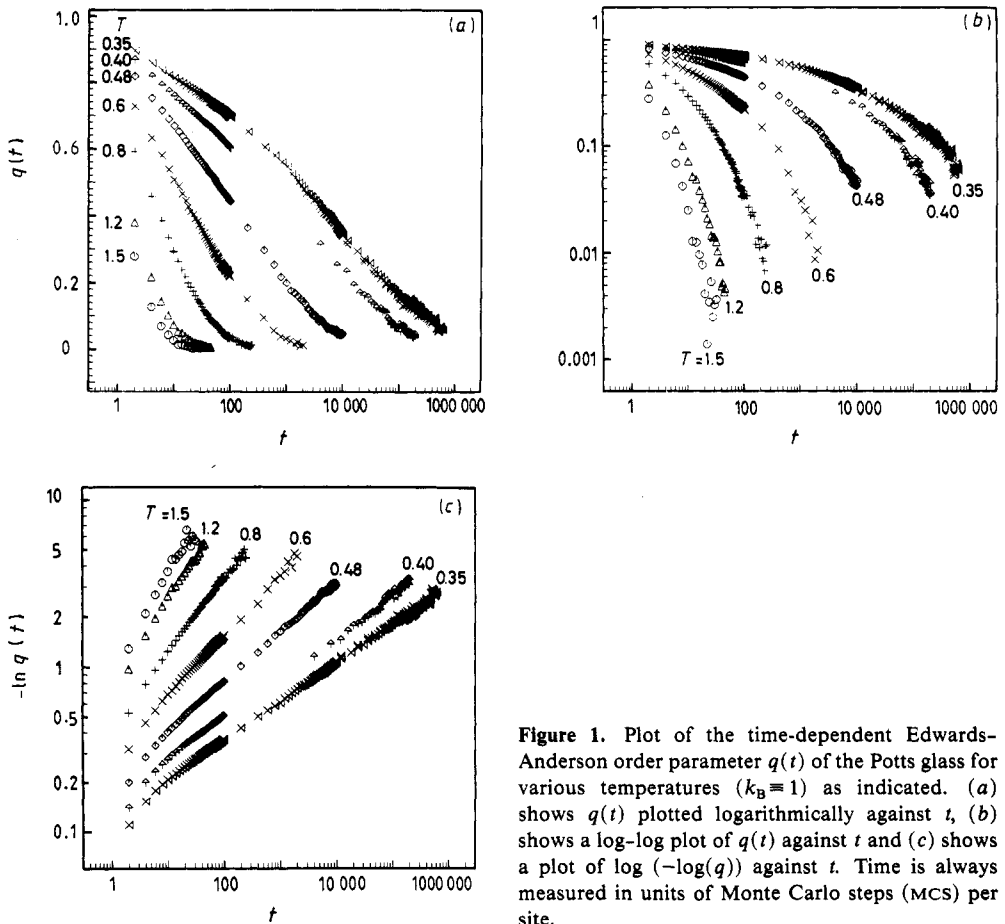


Figure 1. Plot of the time-dependent Edwards-Anderson order parameter $q(t)$ of the Potts glass for various temperatures ($k_B = 1$) as indicated. (a) shows $q(t)$ plotted logarithmically against t , (b) shows a log-log plot of $q(t)$ against t and (c) shows a plot of $\log(-\log(q))$ against t . Time is always measured in units of Monte Carlo steps (MCS) per site.

spin glasses (Binder and Young 1986):

$$q(t) \propto \text{constant} - \log t \quad (2.3a)$$

$$q(t) \propto t^{-\alpha} \quad (2.3b)$$

$$q(t) \propto \exp[-(t/\tau)^y] \quad (2.3c)$$

$$q(t) \propto t^{-\alpha} \exp[-(t/\tau)^y] \quad (2.3d)$$

and (Van Hemmen and Sütö 1985, Van Hemmen and Nieuwenhuys 1986):

$$q(t) \propto (t/\tau)^{-A[\log(t/\tau)]^y}. \quad (2.3e)$$

Of course, all these decay laws are singular for $t \rightarrow 0$ and cannot describe the relaxation of $q(t)$ at short times; in addition, it is clear that (2.3a) cannot hold for late times, where $q(t) \rightarrow 0$, and can therefore at best be a description for intermediate times. Nevertheless, we proceed to test the validity of (2.3a). Figure 1(a) shows that a plot of $q(t)$ against $\log t$ everywhere exhibits some curvature. The strong curvature of the log-log plot (figure 1(b)) shows that (2.3b) is not a reasonable description of the Monte Carlo results either. As the approximate linearity of figure 1(c) shows, the Kohlrausch (1847) law, equation (2.3c), represents a reasonable fit to the data and hence will be used as a phenomenological basis to analyse our data.

Careful work with a special purpose computer has revealed that the three-dimensional Ising spin glass (i.e. the case $p = 2$ in equation (2.1)) can be better described by (2.3d) than by (2.3c) (see Ogielski 1985). One might then expect that (2.3d) should also apply for the case $p = 3$. If such a power law correction actually occurs here as well, the exponent α is presumably smaller than its counterpart in the Ising $\pm J$ spin glass at comparable temperatures. It is also possible, of course, that there is a distinction in the relaxation behaviour between models with a discrete $\pm J$ distribution of bonds, as studied by Ogielski (1985), and the continuous distribution, (1.2), as used here and in Carmesin and Binder (1987a, b). Since we only have accurate data over a somewhat smaller time range than are available for the Ising case, we have not attempted to fit with (2.3d). Similarly, a recent analysis of very precise experimental results on spin glasses has shown that (2.3e) is distinctly preferable to (2.3c, d) (see Van Hemmen and Nieuwenhuys 1986). Unfortunately, the present numerical results lack the very high precision needed to establish (2.3e) in the present case. Both (2.3d, e) involve one parameter more (α or A , respectively) than does (2.3c). Thus the parameters τ and y obtained from fitting the time-dependent Edwards-Anderson order parameter $q(t)$ with (2.3c) should be taken as phenomenological constants providing a numerical representation of $q(t)$, and not necessarily having a deep physical significance.

Nevertheless, it is interesting to note that the exponent y (figure 2) varies approximately linearly with temperature over a wide temperature range. A similar behaviour was in fact noted by Ogielski (1985) for the $\pm J$ Ising spin glass. Ogielski (1985) suggested that at the transition temperature $y \approx \frac{1}{4}$. As we shall see later, in the present case y is somewhat smaller at the transition temperature, if a finite-temperature transition occurs at all.

We now turn to the discussion of the effective relaxation time τ obtained from the fit to (2.3c). Figure 3(a) shows that for $T \leq 1$, where the results are most reliable, a simple Arrhenius behaviour is not consistent with our results, although at low temperatures ($T < 1$) the results could be fitted to an Arrhenius law, $\ln \tau \propto 1/T$. Anyway, no conclusive inference about simple Arrhenius behaviour can be made from this fit (figure 3(a)) because the fitted relaxation times for $T \leq 0.3$ are so large that they distinctly exceed the observation times available in the simulation. Hence we cannot

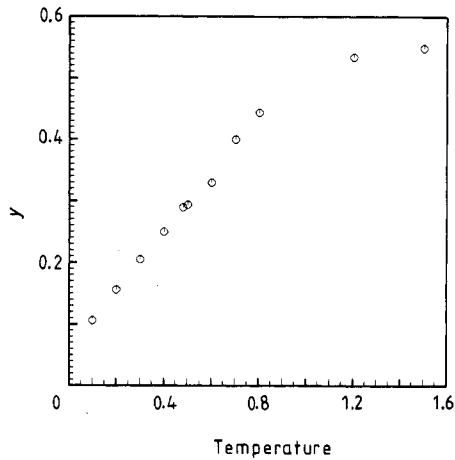


Figure 2. Plot of the effective exponent γ against temperature, as obtained from a fit of results such as shown in figure 1, invoking the validity of equation (2.3c).

consider the estimates of τ for $T \leq 0.3$ to be reliable. Furthermore, our results on the static correlations (see § 3) suggest that our results for $T \leq 0.3$ are also strongly affected by finite-size effects. As a consequence, alternatives to the Arrhenius behaviour should also be considered. If we assume that there is no phase transition at non-zero temperatures and that $\tau \propto \xi^z \propto T^{-\nu z}$, with νz some exponent, we would expect a plot of $\ln \tau$ against $\ln T$ to be linear, with the slope being given by νz at low enough temperatures, such that the asymptotic scaling regime is reached. However, figure 3(d) shows that there is no significant regime where this plot is linear, for the temperatures $T > 0.3$ for which we believe the relaxation time estimates to be reliable, as well as for temperatures $T < 0.3$ for which our estimates for τ are underestimated. If, for $T \approx 0.1$ a straight line fit were possible, the resulting exponent $\nu z \approx 24.8$ is so large that it is clear that the law $\tau \propto T^{-\nu z}$ is not appropriate. In fact, it is more likely that either $\nu z \rightarrow \infty$ (if $T_c = 0$) or one has a phase transition at non-zero temperature. Since the latter behaviour is now believed to occur for the Ising spin glass (see, e.g., Ogielski (1985) or Bhatt and Young (1987)) we have tried a fit to $\tau \propto (1 - T_c/T)^{-\nu z}$ as well. Figure 3(b) shows that choosing T_c to have a very small value ($T_c = 0.11$) the log-log plot still exhibits a significant curvature, while for $T_c = 0.23$ (figure 3(c)) a straight line fit is obtained for a broad temperature range ($0.2 \leq 1 - T_c/T \leq 0.8$). However, since all these temperatures are outside the critical region, it is not clear whether this fit is relevant. In any case, these plots indicate that T_c is smaller than in the corresponding Ising spin glass and that $z\nu$ is larger (remember $z\nu \approx 7$ for the case of the Ising spin glass (Ogielski 1985)). This is not unexpected. This very strong critical slowing down necessarily makes it very difficult to obtain reliable Monte Carlo results both for static and for dynamic quantities, except for temperatures very far from T_c . Therefore ambiguities in the interpretation of the results are inevitable.

3. Static properties

In § 2 we have seen that the relaxation time increases dramatically for $T < \frac{1}{2}$, and hence particular care must be exerted in equilibrating the system, because otherwise all

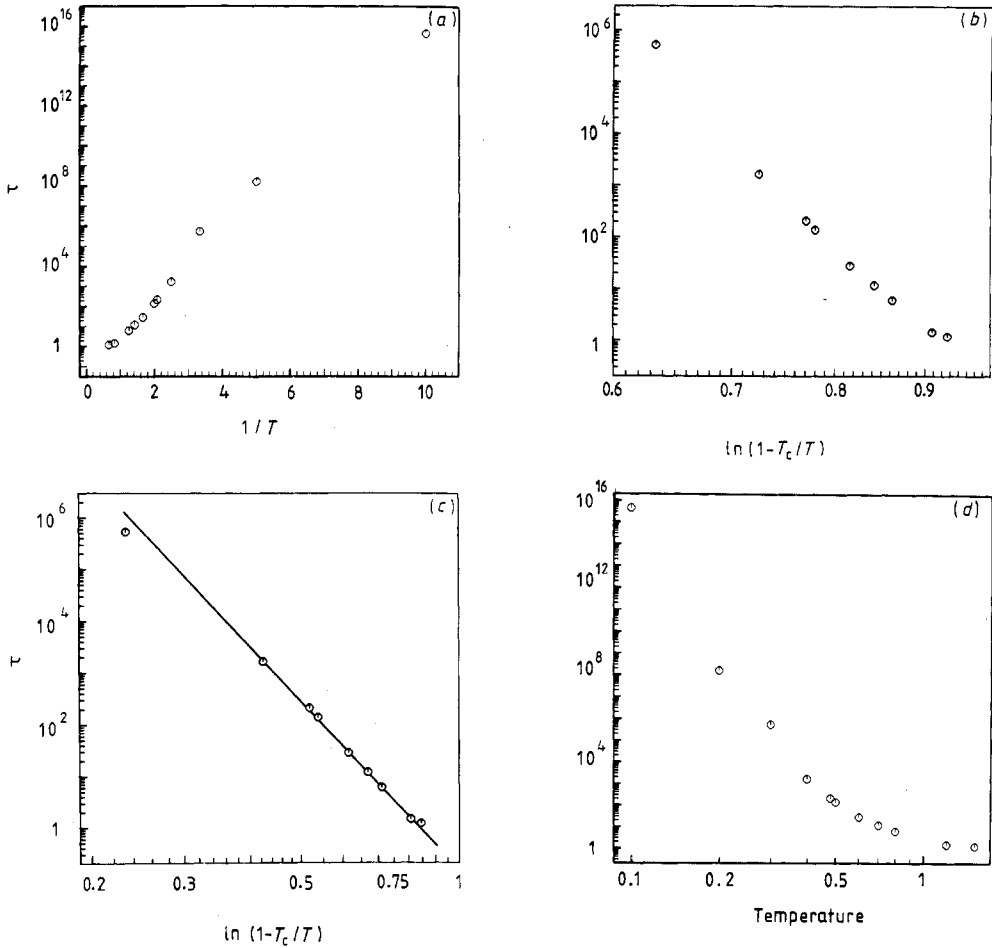


Figure 3. Temperature dependence of the effective relaxation time τ extracted from a fit of $q(t)$ to the Kohlrausch law, equation (2.3c), at various temperatures. (a) presents a plot of $\ln \tau$ against $1/T$ while (b) and (c) show plots of $\ln \tau$ against $\ln(1 - T_c/T)$ with $T_c = 0.11$ and 0.23 , respectively, as well as against $\ln T$ (d).

observations of 'static' properties may be systematically in error due to incomplete equilibration.

Following the pioneering work by Grest *et al* (1986) we studied some quantities by varying the cooling rate: we lower the temperature continuously during the time interval $[0, \Delta\tau]$, $\{1/T(t) = (t/\Delta\tau)(1/T)\}$, to equilibrate the system at the final temperature T . Varying the time constant $\Delta\tau$ of the cooling process over a wide range, we can check whether the results are independent of $\Delta\tau$ for sufficiently large $\Delta\tau$. We find that this is the case for $T \geq \frac{1}{2}$, choosing $\Delta\tau$ in the range from $\Delta\tau = 50$ to $\Delta\tau = 300\,000$ MCS/site. On the other hand, in the temperature region where, possibly, a phase transition occurs ($0.1 \leq T \leq 0.4$) we find that the internal energy E is consistent with a variation

$$E(\Delta\tau) = E(\infty) + \text{constant}/\ln \Delta\tau \quad (3.1)$$

i.e. the same law as proposed by Grest *et al* (1986) for the ground-state energy of the

Ising spin glass. The behaviour described by (3.1) is not a proof that a phase transition has occurred at $T \approx 0.4$, however, since we also find the same behaviour for the isotropic quadrupolar glass at low temperatures (see figure 4), where we expect $T_c = 0$ (see Carmesin and Binder 1987a, b). The behaviour, described by (3.1), has also been found by Grest *et al* for the two-layer model, which has $T_c = 0$, and thus does not require $T_c \neq 0$ but $\tau = \infty$. We rather feel that (3.1) may hold for $T > T_c$ also for a restricted range of $\Delta\tau$ (e.g. $\Delta\tau < \tau$) and therefore the extrapolation according to (3.1) may also be systematically in error: if the equilibrium energy $E(\infty)$ is basically already reached for $\Delta\tau \approx \tau$, (3.1) leads to a systematic underestimation of $E(\infty)$. Thus it is conceivable that the true equilibrium energies lie between the extrapolated values and the values obtained by standard methods where the system is brought to the desired temperature immediately, and the first $\Delta\tau' > \tau$ configurations are discarded from the averaging (see figure 4). Thus we can locate only rather roughly the position of the specific heat maximum, where the E against T curve has its inflection point. For the Potts glass, for which the specific heat $C(T \rightarrow 0) = 0$ due to the discrete nature of the degrees of freedom (the probability of having a degenerate ground state is zero; see Schwartz and Sherrington (1985)), the maximum occurs around $T \approx 0.4$ (see figure 4(d)). For the isotropic quadrupolar glass, where $C(T \rightarrow 0) = 1$ (the model is classical and has two orientational degrees of freedom) the maximum probably occurs near $T \approx 0.2$. In both cases the specific heat maximum is not related to the phase transition, as expected from the experience with spin glasses (Binder and Young 1986).

The problem that in the temperature region of interest static quantities still depend on the cooling rate and need to be extrapolated is not just a problem restricted to the internal energy, but occurs for other quantities as well. As an example, figure 5 presents the response $\langle S_z^2 \rangle$ to a non-zero ordering field $h = 0.1$ in (2.1). Since in the absence of this ordering field $\langle S_x^2 \rangle = \langle S_y^2 \rangle = \langle S_z^2 \rangle = \frac{1}{3}$, the quantity to consider is $\langle S_z^2 \rangle - \frac{1}{3}$. Again, we find that for $T \geq \frac{1}{2}$ there is no difficulty in measuring the equilibrium response, while for $T \leq 0.4$ some cooling rate dependence is detected. For low temperatures ($T \leq 0.3$) the response is nearly T independent, at least within our accuracy. Since we again expect that the resulting value, $\langle S_z^2 \rangle - \frac{1}{3} \approx 0.1$, may overestimate the true result if the curves $\langle S_z^2 \rangle - \frac{1}{3}$ against $1/\ln \Delta\tau$ bend over horizontally at smaller $1/\ln \Delta\tau$ where $\Delta\tau \approx \tau$, we can at least conclude that there is no tendency to establish a ferromagnetically ordered Potts phase at low temperatures, although for $1 < T < 10$ the variation of $\langle S_z^2 \rangle - \frac{1}{3}$ with T is steeper than according to the Curie law. At this point, we recall that mean-field theory for the infinite-range Potts glass (Gross *et al* 1985) even predicts two successive phase transitions: as the temperature is lowered, one first encounters a transition from the disordered phase to a glass phase, while at lower temperatures a transition to a different type of glass phase occurs. There are no observations whatsoever that would indicate that the short-range system also has two successive phase transitions.

Figure 6 shows our results for the correlation function and correlation length, which we have defined as in the quadrupolar case (Carmesin and Binder 1987a, b)

$$g(\mathbf{R}) = \left(\left\langle \left\langle \left(S_i \cdot S_j \right)^2 - \frac{1}{p} \right\rangle^2 \right\rangle_{\substack{i, j \\ \mathbf{R}_i - \mathbf{R}_j = \mathbf{R}}} \right)_{\text{av}} p^2 (p-1)^{-1} \tag{3.2a}$$

$$1/\xi \approx -\ln g(\mathbf{R})/R \tag{3.2b}$$

choosing the direction of \mathbf{R} to lie in a lattice direction. Here $\langle \dots \rangle_{\mathbf{R}_i - \mathbf{R}_j = \mathbf{R}}$ describes the average over those pairs of sites for which $\mathbf{R}_i - \mathbf{R}_j = \mathbf{R}$ holds. Again, the results

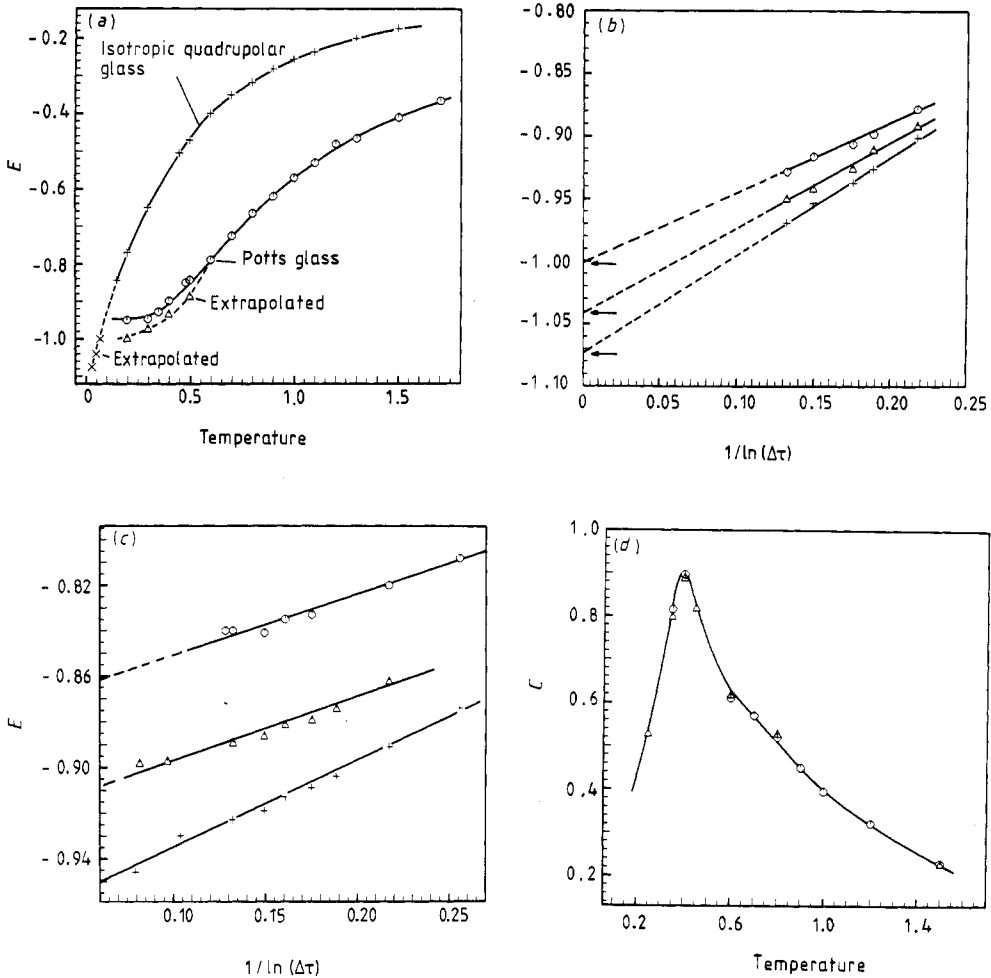


Figure 4. (a) Internal energy E per site for the Potts glass and the isotropic quadrupolar glass (with vector dimensionality $m = 3$) plotted against temperature. (b) and (c) Extrapolation according to (3.1) for the isotropic quadrupolar glass (b) and the Potts glass (c); the extrapolated values (arrows) are included as triangles (or lying crosses, respectively) in the main figure, while octagons (standing crosses) are due to the standard method. Temperatures shown are $T = 0.075$ (octagons), 0.05 (triangles) and 0.025 (crosses) in (b), and $T = 0.4$ (octagons), 0.3 (triangles) and 0.2 (crosses) in (c). (d) Specific heat C of the Potts glass plotted against temperature. Octagons are data obtained from energy fluctuations, while triangles are obtained from numerical differentiation of the E against T curve. A curve is drawn only to guide the eye.

for the lowest temperatures shown may suffer somewhat from systematic errors due to incomplete equilibration. But the results clearly show that the correlation length is indeed rather small in this temperature range, $\xi \approx 2$ lattice spacings. Thus with $L = 12$ and $L = 18$ finite-size effects should not be a problem. A log-log plot of ξ against $1 - T_c/T$ for the choice of T_c where the relaxation time was reasonably well fitted, $T_c \approx 0.23$, still exhibits some curvature, indicating that possibly T_c is even lower (figure 6(c)). However, for the choice $T_c = 0$ the curvature goes in the other direction, indicating that T_c might be non-zero (figure 6(b)). The slope of the straight line in figure 6(c)

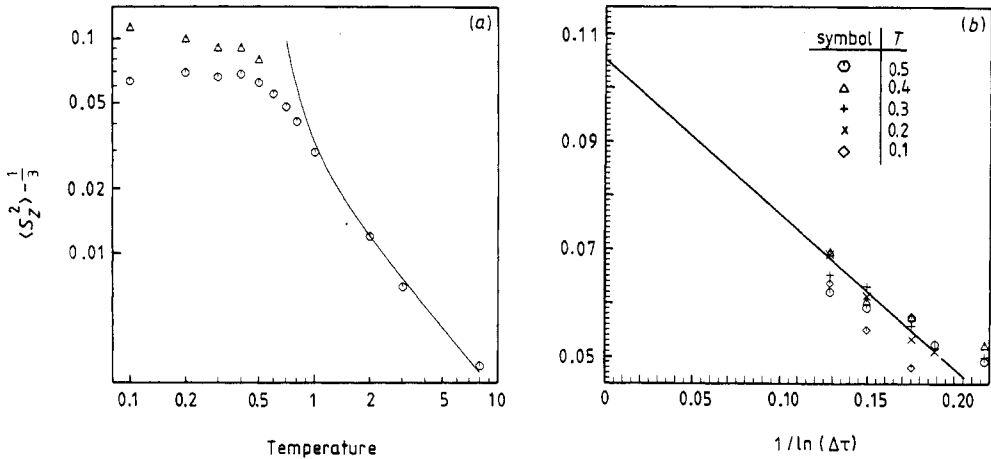


Figure 5. Temperature dependence of $\langle S_z^2 \rangle^{-1/3}$ at $h = 0.1$ plotted on log-log scales (a) and extrapolation of $\langle S_z^2 \rangle^{-1/3}$ against $1/\ln(\Delta\tau)$, (b). Octagons in the upper part are obtained from linear extrapolation, as shown in (b), while triangles are equilibrium results obtained in the standard way (see figure 4). The line represents the result obtained by a high-temperature expansion.

implies $\nu \approx 2$, while figure 6(c) implies that ν is as least as large as $\nu \approx 2.2$. But the small values of ξ indicate that the asymptotic critical region has not been reached and thus these results do not exclude rather different values for the critical exponent ν .

In view of the foregoing it is also no surprise that the ordering susceptibility

$$T\chi \equiv \sum_{\mathbf{R}} g(\mathbf{R}) \tag{3.3}$$

also cannot be interpreted unambiguously (figure 7). While for $T_c = 0$ the log-log plot exhibits pronounced curvature throughout the temperature regime shown, and so only a lower bound for the exponent $\gamma\{T\chi \propto T^{-\gamma}\}$ could be derived: $\gamma \geq 4.75$. Reasonable straight line behaviour occurs for choices of non-zero values of T_c , such as $T_c = 0.23$ or $T_c = 0.11$. The resulting choices of γ ($\gamma \approx 3.8$ or $\gamma \approx 12.6$ respectively) satisfy the inequalities $\gamma \leq 3\nu$ and $\gamma \leq z\nu$, and thus these exponent estimates at least are not completely unreasonable. But again the results do not yield any clear preference for the choice of T_c .

Figure 7 also shows that for $T \geq 1.0$ the results for χT are rather well described by the first terms of a systematic high-temperature expansion (Carmesin and Ohno 1987). Extending such series to high order, as in the Ising case (Singh and Chakravarty 1986), should hence yield valuable additional information on this model. One has to keep in mind, however, that below the corresponding ferromagnetic transition temperature it is still an open question whether such expansions converge at all.

4. The three-dimensional short-range Potts glass: a system at its lower critical dimensionality?

From the analysis of §§ 2 and 3 it is clear that the short-range Potts glass model Hamiltonian ((1.1) and (1.2)) leads to a dramatic increase of the relaxation time as

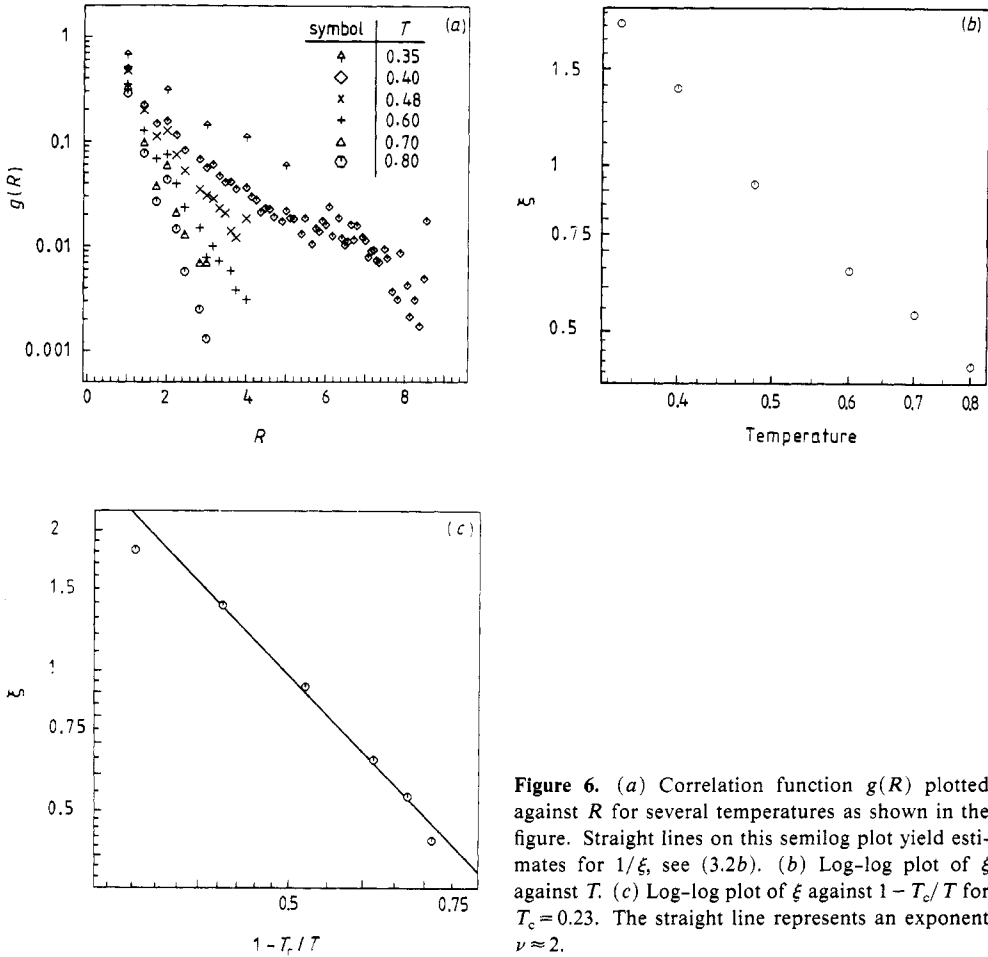


Figure 6. (a) Correlation function $g(R)$ plotted against R for several temperatures as shown in the figure. Straight lines on this semilog plot yield estimates for $1/\xi$, see (3.2b). (b) Log-log plot of ξ against T . (c) Log-log plot of ξ against $1 - T_c/T$ for $T_c = 0.23$. The straight line represents an exponent $\nu \approx 2$.

the temperature is decreased, and hence a reliable estimation of the phase transition behaviour of the model is very difficult. Nevertheless our results suggest the following tentative conclusions.

(i) If the lower critical dimension of the model exceeds three, one would expect simple power laws for the relaxation time, correlation length and susceptibility: $\tau \propto T^{-\nu_z}$, $\xi \propto T^{-\nu}$, $\chi T \propto T^{-\gamma}$. However, the results clearly do not suggest such a behaviour: these laws, if they hold at all, could hold only for the lowest temperatures studied and all the lower bounds obtained on the exponents ν_z , ν and γ are so large that it is rather likely these exponents are infinite, i.e. $d = 3$ would be the lower critical dimension.

(ii) If $d = 3$ exceeds the lower critical dimension, one should have a non-zero critical temperature T_c and the temperature variable in the above power laws is $1 - T_c/T$ rather than T . Our Monte Carlo results are consistent with such a behaviour, but unfortunately a broad range of values for T_c yields fits of similar quality and it is clear that the data do not fall in the critical region $(1 - T_c/T) \leq 0.1$.

Can our results at least be taken as evidence that there is a non-zero T_c ? We feel that even this conclusion is doubtful, for the following reason. If $T_c = 0$ and the system is at the lower critical dimension, as inferred above, we would expect a behaviour

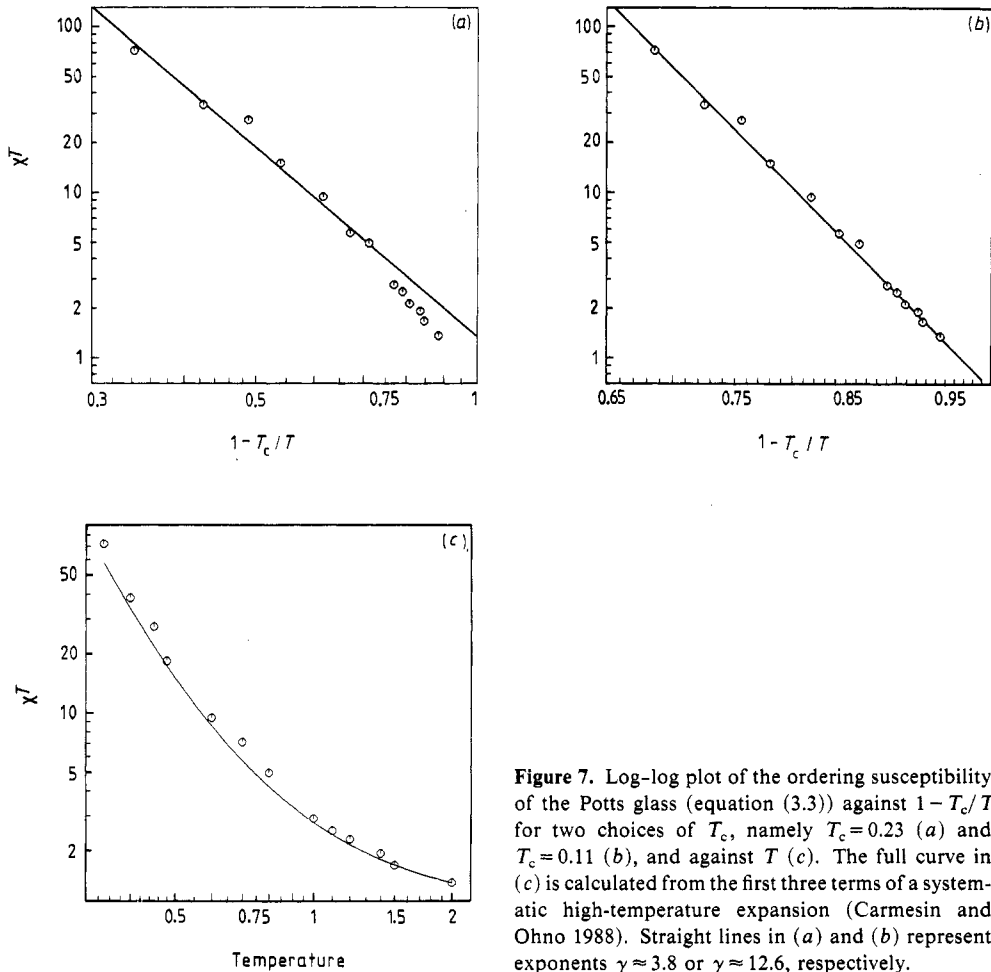


Figure 7. Log-log plot of the ordering susceptibility of the Potts glass (equation (3.3)) against $1 - T_c/T$ for two choices of T_c , namely $T_c = 0.23$ (a) and $T_c = 0.11$ (b), and against T (c). The full curve in (c) is calculated from the first three terms of a systematic high-temperature expansion (Carmesin and Ohno 1988). Straight lines in (a) and (b) represent exponents $\gamma \approx 3.8$ or $\gamma \approx 12.6$, respectively.

described by exponential divergences

$$\ln \xi \propto T^{-\nu'} \quad \ln \tau \propto T^{-z'} \quad \chi T \propto \xi^{\gamma'} \quad (4.1)$$

where the exponents ν' , z' and γ' are finite (McMillan 1984, Bray and Moore 1987). To test (4.1) we present log-log plots of $\ln \tau$ and $\ln \chi T$ against T (figure 8). It is seen that both τ and χT are consistent with (4.1), with $\nu' \approx 1.2$ and $z' \approx 2.1$.

On the other hand, a phenomenological renormalisation treatment of spin glasses (McMillan 1984) yields $\nu' = 2$, $z' = \nu' + 1$ and hence $z' = 3$. While the values we obtained for these exponents agree well with $z' = \nu' + 1$, they clearly disagree with $\nu' = 2$ and hence they disagree with $z' = 3$ as well. It is not clear to us whether the phenomenological renormalisation results for z' and ν' should be considered rigorous. But we fully agree with the point made by Bhatt and Young (1987) who stated that for spin glasses several methods needed to be combined (finite-size scaling methods and 'domain wall renormalisation group methods', in addition to straight Monte Carlo as presented here), in order to locate the phase transition convincingly: such additional methods will be needed here as well. Also, the extension of the present study to larger numbers, p , of Potts states would be very interesting. And an investigation of other spatial

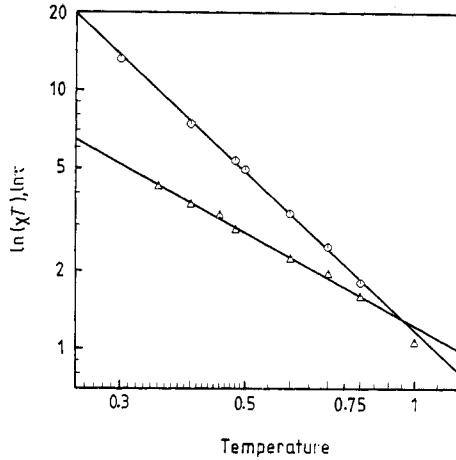


Figure 8. Log-log plot of $\ln \tau$ (triangles) and $\ln(\chi T)$ (octagons) against temperature. Straight lines indicate the behaviour given by (4.1), with $\nu' \approx 1.2$ and $z' \approx 2.1$.

dimensionalities ($d = 2$, $d = 4$) will be needed to further elucidate the problem of the lower critical dimension.

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